BEAM-BUNCHING WITH A LINEAR-RAMP INCLUDING SPACE-CHARGE FORCE EFFECTS CYLINDER MODEL

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Abstract

The voltage-amplitude requirement of a saw-tooth waveform buncher is calculated to give a desired degree of bunching for a given beam current and particle species. This calculation includes the effect of space-charge forces with and without adjacent beam buckets. The results are compared to TRACE-3D calculations which do not include the space-charge effects of adjacent bunches. It appears that TRACE-3D calculations underestimate the bunching voltage required. The methodology and a listing of the spread sheet that performs the analytical bunching calculation are included.

Models

The beam consists of a series of uniform-density charge cylinders, see Fig. 1, that are spaced $D=\beta\lambda$ apart, with length $L=D\varphi/180$ where φ is half the total bunched-beam phase spread, β is the average beam velocity divided by the velocity of light (c), and λ is the fundamental-frequency-rf free-space wave length.

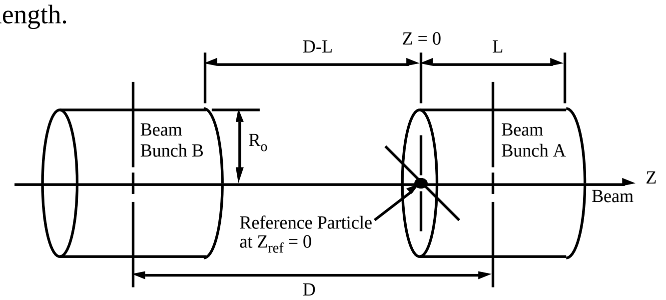

Fig. 1. Geometry for the beam bunching calculation. The motion of a reference particle at the edge of beam bunch A is determined. Two different equations of motion which include effects of space-charge forces from beam bunch A but with and without space-charge forces from beam bunch B are compared.

Space-charge forces, due to beam buckets adjacent to the reference particle, are calculated for a reference particle that is on the beam axis at a cylinder edge. The motion of this reference particle models the bunching of the entire distribution. The electrostatic potential for the reference particle is calculated from which the axial electric field seen by this particle is determined. The equation of motion for the axial motion of the reference particle is solved (the on axis reference particle experiences no transverse forces) which gives the required bunching voltage in the beam center-of-mass frame of reference. Velocities are added nonrelativistically to give the buncher voltage in the laboratory reference frame.

TRACE-3D uses a uniformly filled ellipsoidal model to estimate the space-charge forces. The equation of motion for an on-axis reference particle, using the ellipsoidal model, is integrated and the resulting bunching voltage is compared to TRACE and the cylinder model.

Derivation - Cylinder Model

Given that I is the beam current and f is the bunch frequency, the charge density of the beam in the cylinder is

$$\rho = I \big/ \left(\pi R_0^2 L f\right) \tag{1}$$

where R_o is the beam radius (R_o is assumed to remain constant during the bunching process).

We consider only electrostatic forces in the Lorentz force equation for the time evolution of the reference particle, ignore all image-charge forces that could exist due to a beam pipe, and consider only axial motion. Therefore,

$$m\frac{dv}{dt} = eE_z \ . \tag{2}$$

where m is the particle mass, e is the electron charge, E_z is the axial electric field due to space charge, t is time, and v is the velocity of the reference particle in the beam bunch rest frame.

The on-axis axial electric field due to the two bunches can be obtained from the potential function

$$\Phi = \frac{1}{4\pi\varepsilon_o}\left(\int_{D}^{D+L} dZ \int_0^{R_0} \frac{\rho}{\sqrt{\left(Z - Z_{ref}\right)^2 + R^2}}\, R\,dR \int_0^{2\pi} d\theta + \int_{D}^{L} dZ \int_0^{R_0} \frac{\rho}{\sqrt{\left(Z - Z_{ref}\right)^2 + R^2}}\, R\,dR \int_0^{2\pi} d\theta \right) \tag{3}$$

where ε_o is the free space permitivity. The first set of integrals is for bunch B, and the second set for bunch A. Integrating Eq. (3) for Φ, taking the derivative with respect to Z_{ref} (Z_{ref} is the Z coordinate of the reference particle), and setting Z_{ref} equal to zero to obtain E_z gives

$$E_z(\text{bunch A}) = \frac{\rho}{2\varepsilon_0}\left(\sqrt{L^2 + R_0^2} - R_0 - L\right) \tag{4}$$

for the electric field seen by the reference particle due to bunch A, and

$$E_z(\text{bunch B}) = \frac{\rho}{2\varepsilon_0}\left[\sqrt{\left(D-L\right)^2 + R_0^2} - \sqrt{D^2 + R_0^2} + L\right] \tag{5}$$

for the electric field due to bunch B. Adding Eqs. (4) and (5) gives the total electric field seen by the reference particle

$$E_{zT} = \frac{\rho}{2\varepsilon_0}\left[\sqrt{L^2 + R_0^2} - R_0 - \sqrt{D^2 + R_0^2} + \sqrt{\left(D-L\right)^2 + R_0^2}\right].$$

Because the electric field does not depend on velocity, Eq. (2) can be integrated to give the required reference-particle energy gain, δW_{cm}, due to bunching in the beam rest frame,

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$$\frac{\delta W_{cm}}{e} = \frac{m}{2e}\int_{v_0}^{0} dv^2 = \frac{1}{2}\left(\frac{mc^2}{e}\right)\beta^2 = \int_{L=D}^{L_{\min}} E_{zT}\, dL\,. \qquad (7)$$

The initial velocity v_o for an unbunched beam ($L = D$) gives a final bunch length L_{min} when $v = 0$. The bunch length, L, and the beam radius, R_o, are normalized to the bunch center separation distance, D, by defining $r = R_o/D$ and $s = L/D$. Using Eqs. (1), (4), (5), and (6) in Eq. (7) and integrating gives

$$\delta W_{cm} = \frac{eI}{2\pi r^2 f \varepsilon_o D}\left(J_{\mathrm{A}} + J_{\mathrm{B}}\right) \qquad (8)$$

where

$$J_A = \sqrt{s^2+r^2} - \sqrt{1+r^2} + r\ln\left(\frac{r+\sqrt{1+r^2}}{r+\sqrt{s^2+r^2}}\right) + 1 - s \qquad (9)$$

and

$$J_B = \left[\begin{array}{l} s+\sqrt{(1-s)^2+r^2} \\ -\sqrt{1+r^2}\,\ln\left(\dfrac{\sqrt{1+r^2}\,\sqrt{(1-s)^2+r^2}+1+r^2-s}{r\left(r+\sqrt{1+r^2}\right)}\right) \\ -\ln\left(\dfrac{\sqrt{(1-s)^2+r^2}+s-1}{r}\right)-r-1 \end{array}\right]. \qquad (10)$$

Velocities, corresponding to the center-of-mass bunching energy-spread and the average beam velocity, are added to give the buncher voltage required in the laboratory reference-frame. The nominal beam-velocity in the laboratory reference frame is $v_0 = \sqrt{2W_0/m}$ where W_o is the nominal beam energy. The velocity of the reference particle in the beam center-of-mass reference frame is $\delta v = \sqrt{2\delta W_{cm}/m}$. Calculating the reference particle's energy in the laboratory reference frame and subtracting the average beam energy gives the energy gain that the buncher must supply to the reference particle which is

$$\delta W_{lab} = \delta W_{cm} + \sqrt{4W_0\,\delta W_{cm}}\,. \qquad (11)$$

Derivation - Ellipsoidal Model

The electric field due to a uniform-charge-density-ellipsoid beam-bunch seen on axis by a reference particle at the edge of a single bunch is [1], [2], [3]

$$E_z = \frac{3IZ_{ref}}{4\pi\varepsilon_o f R_0^2 (L/2)}\, g(p)\,,\quad p = \frac{(L/2)}{R_0}\,, \qquad (12)$$

and g is a "form factor" which can be approximated by [1]

$$g(p) = 1/(3p). \qquad (13)$$

Substituting Eq. (13) into Eq. (12) and picking the reference particle coordinates to be on axis at the beam edge (R=0, Z_{ref}=L/2) gives for the electric field

$$E_z = \frac{I}{4\pi\varepsilon_o f R_0 Z_{ref}}\,. \qquad (14)$$

Carrying through the same procedure as for the cylinder model gives

$$\delta W_{cm} = \frac{eI}{4\pi\varepsilon_o r f D}\ln\left(\frac{1}{s}\right). \qquad (15)$$

Combining Eqs. (11) and (15) gives the buncher voltage required in the Lab system.

Examples and Conclusion

Figure 2 show the spread sheet used to calculate the buncher voltage. The parameters are meant to be self explanatory. Figure 3 shows a comparison of this model calculation to results obtained from TRACE-3D, which uses an ellipsoidal beam bunch model. The TRACE transport channel, used for the comparisons, consisted of a periodic series of solenoid magnets with the beam channel focusing strength set to minimize the space-charge tune depression even in the maximum bunching case. For the maximum bunching case, the initial beam size was increased 3% to keep the beam nearly matched (the solenoid magnetic field strength was not varied).

The discrepancy between the TRACE calculation and the spread sheet calculation for the ellipsoidal distribution is due to the approximation used for the form factor in Eq. (13) where this approximation overestimates the space-charge force by as much as 10%. The difference between the cylindrical model and the ellipsoidal model can be understood by comparing the ratio of the electric field calculated in Eqs. (4) and (14). Taking the ratio of these two equations and using Eq. (1) gives

$$\frac{\text{Eq. (4)}}{\text{Eq. (14)}} = 2\left(1 - \frac{R_o}{2L}\right) \qquad (16)$$

Equation (16) gives a value of close to 2 for the ratio corresponding to our examples. This ratio is also the ratio of the center-of-mass energy spread required for bunching. Using Eq. (11) to transform to the Lab frame shows that the buncher voltage required for the cylinder model should be 40% ($\sqrt{2}$) higher than the ellipsoidal model and is consistent with the result shown in Fig. (3). Also, the TRACE beam distribution for $\phi = 180^{\circ}$ is already bunched with a pseudo-gaussian shape which causes a further underestimate of the buncher voltage required to obtain the final degree of bunching. Figure 3 shows that including adjacent bunches for calculating longitudinal space-charge effects is important only for minimal bunching ($\phi \geq 150^{\circ}$).

BUNCHING DESIGN SPREAD SHEET 8/24/95 9:52

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ACHRONYM	PARAMETER	UNITS	EQUATION	VALUES
	CONSTANT PARAMTERS			
ε	Permittivity of free space	F/m	fixed constant	8.8540E-12
π	Pi	none	fixed constant	3.1416E+00
c	Velocity of light	m/s	fixed constant	2.9979E+08
	INPUT PARAMTERS			
M	Particle rest-mass energy	MeV	input	9.3800E+02
Q	Particle charge number	none	input	1.0000E+00
W	Beam energy	MeV	input	7.5000E-01
f	Fundamental bunching frequency	Hz	input	2.0125E+08
I	Beam current	A	input	2.0000E-02
R	Beam radius	m	input	3.3000E-03
ϕ	(+/-) final desired bunching phase spread	deg	input	4.6000E+01
	KINEMATIC PARAMETERS ETC.			
γ	Relativistic gamma	none	(M+W)/M	1.0008E+00
β	Relativistic beta	none	SQRT((γ ^2−1)/γ^2)	3.9965E-02
P	Beam momentum	MeV/c	βγM	3.7517E+01
D	Length between bunch centers	m	β*c/f	5.9534E-02
r	Normalized bunch radius	none	R/D	5.5430E-02
s	Desired normalized bunch length	none	ϕ/180	2.5556E-01
	ALGEBRA			
J1	Integral parameter (cylinder bunch calculation)	none	SQRT(s^2 + r^2)	2.6150E-01
J2	Integral parameter (cylinder bunch calculation)	none	SQRT(1+r^2)	1.0015E+00
J3	Integral parameter (cylinder bunch calculation)	none	SQRT[(1-s)^2+r^2]	7.4651E-01
K1	Multiplication constant	none	Q*I/(2*π*r*2*f*ε*D)	9.7660E+03
JDL-part	Partial integral for E(D-L) (cylinder bunch calc.)	none	J2*ln((J2*J3+J2^2-s)/(r*(r+J2)))	3.2444E+00
JDL	Total integral for E(D-L) (cylinder bunch calc.)	none	(JDL-part)+1-s-J3+r+ln((J3+s-1)/r)	5.7658E-03
J0	Integral for E(Z0) (cylinder bunch calc.)	none	J1-J2+(1-s)+r*ln((r+J2)/(r+J1))	7.1172E-02
Wcm	Adjacent-cylinder-bunch center-of-mass energy	eV	K1*(J0-JDL)	6.3875E+02
Wsing	Single-cylinder-bunch center-of-mass energy	eV	K1*J0	6.9506E+02
Wellipse	Single-ellipse-bunch center-of-mass energy	eV	(K1*r/2)*ln(1/s)	3.6927E+02
	BUNCHER VOLTAGE TO GIVE DESIRED FINAL PHASE WITH SPACE-CH.			
	Buncher voltage for two cylinder bunches	V	Wcm + sqrt(4*W*Wcm)	**4.4414E+04**
	Buncher voltage for single cylinder bunch	V	Wsing + sqrt(4*W*Wsing)	**4.6359E+04**
	Buncher voltage for single ellipsoidal bunch	V	Wellipse + sqrt(4*W*Wellipse)	**3.3653E+04**

Fig. 2. Spread sheet to calculate the required buncher voltage to bunch the beam to a desired phase spread. Calculations are done for both a single bunch and for adjacent beam bunches.

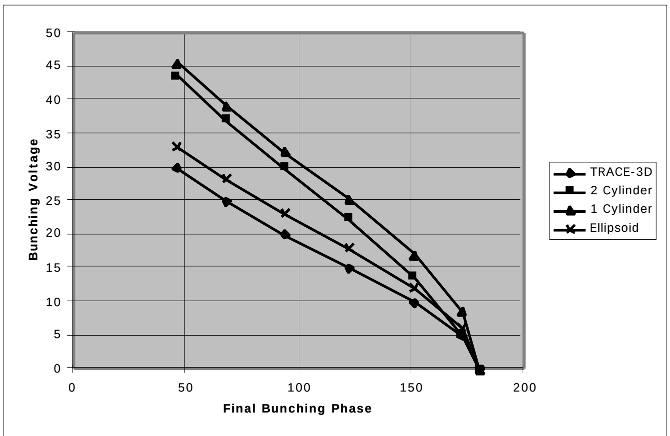

Fig. 3. Comparison of the cylindrical model calculation to results obtained from TRACE-3D for various degrees of bunching for a fixed beam radius.

References

[1] T. P. Wangler, "Space-Charge Limits in Linear Accelerators," LA-8388 (1980).

[2] K. R. Crandall, D. P. Rusthoi, "TRACE 3-D Documentation," LA-UR-90-4146.

[3] O. D. Kellogg, "Foundations of Potential Theory," (1929).